U.S. Department of Transportation
Federal Aviation Administration

Advisory Circular

Subject: MEANS OF COMPLIANCE WITH SECTION 23.629, FLUTTER

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Change:

1. PURPOSE. This advisory circular presents information and guidance to provide one means, but not the only means, of complying with Section 23.629, Flutter (including flutter, airfoil divergence, and control reversal) of Part 23 of the Federal Aviation Regulations. Accordingly, this material is neither mandatory nor regulatory in nature.

2. CANCELLATION. AC 23.629-1, Means of Compliance With FAR 23.629, Flutter, dated January 8, 1979, is cancelled.

3. BACKGROUND. The complexity of the flutter problem has historically prompted endeavors to find simplified methods of flutter substantiation. Although the advent of electronic computers has deemphasized the need to make drastic assumptions previously necessary to enable mathematical treatment of the flutter problem, there remains a need to simplify the flutter problem as much as possible consistent with safety in order to minimize the cost and effort required to show freedom from flutter. Past experiences gained by the necessity to judiciously choose meaningful degrees of freedom, and by the need to make parametric studies to establish practical boundaries of the effectiveness of the various physical quantities, has resulted in a generally recognized set of good practices. These good practices form the basis for this advisory circular.

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CONTENTS

CHAPTER 1. AIRPLANE CATEGORIES

1. GENERAL. Airplanes in the general category are those with a typical exterior configuration; i.e., high, mid, or low wing; single fin and single horizontal stabilizer aft-mounted on the fuselage; and tractor powerplant installations.

2. SPECIAL DESIGN. The special design category includes airplanes with certain design features that experience has shown warrant special consideration with regard to flutter. Flutter free operation for these special unconventional configurations may be shown by analyses which include an assessment of the effects of critical parameters. Flight flutter tests to supplement those analyses are recommended. Some of these special unconventional configurations are:

a. Any aircraft with a design dive speed of 260 knots (EAS) or more at altitudes below 14,000 feet and Mach 0.6 or more at altitudes at and above 14,000 feet.

b. Any aircraft approved for flight in icing conditions. (The effect of ice accretions on unprotected surfaces, including those which might occur during system malfunctions, should be considered).

c. Pusher powerplants.

d. Canard geometry.

e. T, V, X, H, or any other unusual tail configuration.

f. Any external pods or stores mounted to wing or other major aerodynamic surface.

g. Fuel tanks outboard of 50% semispan.

h. Tabs which do not meet the irreversibility criteria of chapter 2, paragraph 3.d., and of reference 1, appendix 4.

i. Spring tabs.

j. All-movable tails, i.e., stabilators.

k. Slender boom or twin-boom fuselages.

l. Multiple-articulated control surfaces.

m. Wing spoilers.

n. Hydraulic control systems with stability augmentation.

o. Full span flaps.

p. Leading edge devices (i.e., slots, etc.).

q. Geared tabs (servo or anti-servo, etc.).

CHAPTER 2. METHODS OF SUBSTANTIATION

3. SIMPLIFIED CRITERIA.

a. Guidelines. Airframe and Equipment Engineering Report No. 45 is intended to serve as a guide to the small airplane (V_D less than 260 knots EAS at altitudes below 14,000 ft.) designer in the prevention of flutter, aileron reversal, and wing divergence. The material presented relies upon:

(1) A statistical study of the geometric, inertia, and elastic properties of those airplanes which had experienced flutter in flight, and the methods used to eliminate the flutter.

(2) Limited wind-tunnel tests conducted with semi-rigid models. These were solid models of high rigidity with motion controlled at the root by springs to simulate wing bending and torsion. Springs at the control surface were used to simulate rotation.

(3) Analytic studies based on the two-dimensional study of a representative section of an airfoil.

b. Wing and Aileron. Prevention of wing flutter is attempted through careful attention to three parameters; wing torsional flexibility, aileron balance, and aileron free play.

(1) The aileron balance criteria is obtained from the aileron product of inertia, K, about the wing fundamental bending node line and the aileron hinge line; and the aileron mass moment of inertia, I, about its hinge line. A limit of the parameter, K/I, is set as a function of V_D.

(2) A wing torsional flexibility factor, F, is defined and a limit established as a function of V_D. In order to apply the criteria, one needs to know wing twist distribution per unit applied torque, wing planform, and limit dive speed.

(3) The total free play of each aileron with the other aileron clamped to the wing must not exceed the specified maximum.

c. Elevator and Rudder. Dynamic balance criteria for the elevator and rudder (similar to the K/I of the aileron) are defined and limits set as a function of limit dive speed. In order to utilize the criteria, the following information is required:

(1) Geometry - horizontal tail semichord at the midspan
- semispan of horizontal tail
- distance from fuselage torsion axis to tip of fin
- semichord of vertical tail measured at 70% span position

(2) Stiffness - Fuselage vertical bending frequency
- Fuselage torsional frequency
- Fuselage lateral bending frequency

(3) Mass - Elevator static balance about hinge line
- Elevator mass moment of inertia about hinge line
- Elevator product of inertia referred to stabilizer centerline and elevator hinge line
- Rudder static balance about hinge line
- Product of inertia of rudder referred to fuselage torsion axis and rudder hinge line
- Rudder mass moment of inertia about hinge line

d. Tabs. It is recommended that all reversible tabs be balanced about the tab hinge line. The degree of static and dynamic balance should be determined by rational analyses (reference chapter 4). In practice, most tabs are irreversible, which means:

(1) For any position of the control surface and tab, no appreciable deflection of the tab can be produced by means of a moment applied directly to the tab when the control surface is held in a fixed position.

(2) The total free play at the tab trailing edge should be less than the following:

(i) If the tab span does not exceed 35 percent of the span of the supporting control surface, the total free play shall not exceed two percent of the distance from the tab hinge line to the trailing edge of the tab perpendicular to the tab hinge line.

(ii) If the tab span equals or exceeds 35 percent of the span of the supporting control surface, the total free play is not to exceed one percent of the distance from the tab hinge line to the trailing edge of the tab perpendicular to the tab hinge line.

(3) The tab natural frequency should be equal to or should exceed the calculated value and expressed as a function of tab and control surface geometry and airplane dive speed. (reference 1).

(4) Spring loaded tabs are free to rotate and thus are not irreversible. Generally, these tabs will require dynamic as well as static balance. Extensive flutter analysis is always needed to define these requirements.

4. RATIONAL ANALYSIS.

a. Review of Past Analysis. Review of previous flutter analyses conducted upon similar aircraft can provide the engineer with useful information regarding trends, critical modes, etc. Although in general such a review is not used as a substantiation basis for a new aircraft, it can provide a useful tool in evaluating the effect of modifications to existing certified aircraft. Chapter 3 provides additional comments on this subject.

b. Two-Dimensional Analysis. The flutter characteristics of straight wings (or tails) of large aspect ratio can be predicted reasonably well by considering a "representative section" that has two or three degrees of freedom. Translation and pitch are always needed and, for control surfaces, the third freedom would be rotation about the hinge line. Appendix 2 presents a more thorough discussion of this approach.

c. Three-Dimensional Analysis. Current analysis is based upon consideration of total span, rather than "representative section" discussed in 4.b. above. The behavior is integrated over the whole structure being analyzed. Some idealization is always necessary; the most common being the division of the span into strips. Other types of modeling are also used. Generalized mathematics are presented in appendix 2.

For Part 23 airplanes, quite often the wing and empennage analyses are conducted separately; however, this is not always adequate for unconventional configurations. Both the symmetric and antisymmetric motions require investigation.

Calculated mass and stiffness distributions are generally used to calculate uncoupled modes and frequencies. These values are then used to conduct a coupled vibration analysis; the resulting coupled modes and frequencies are then usually compared with measured natural modes.

The calculated stiffness-related inputs are generally adjusted until good agreement is obtained with the test data. Once satisfactory agreement is achieved, the coupled vibration analysis is normally used for the flutter calculations.

It is suggested that one perform certain variations in the assumed input conditions to see which parameters are critical. Control surface balance conditions and system frequencies (especially tab frequencies) are often investigated parametrically. The effect of control system tension values at the low and high ends of the tolerance range should be assessed.

It may be advantageous to arbitrarily vary certain main surface frequencies (stiffness), especially torsional frequencies and engine mode frequencies, while leaving other frequencies constant.

Sometimes it is desirable to evaluate the effect of a slight shift in spanwise node location for a very massive item where the node is located very close to or within the item. (Test data may not be sufficiently accurate for this assessment.)

It is normal practice to run a density-altitude check to include near-sea-level, maximum and any other pertinent altitudes such as the knee of the airspeed-altitude envelope where the design dive speed becomes MACH-limited.

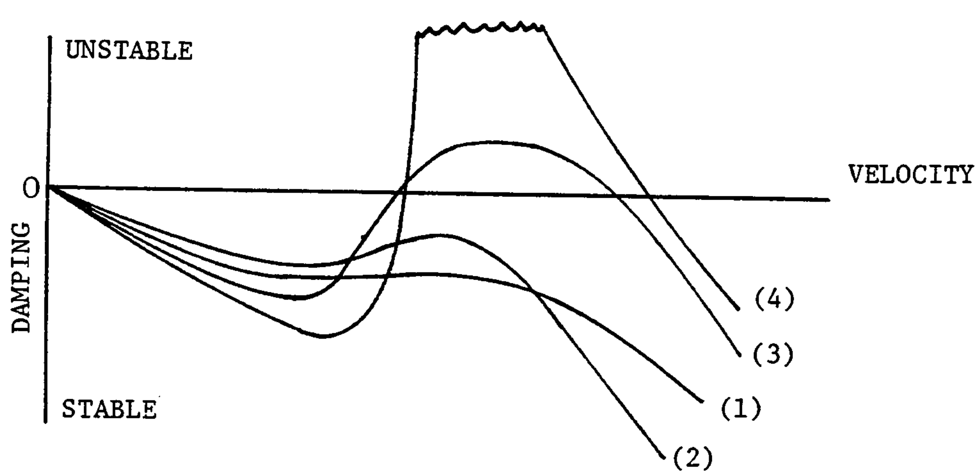
It is desirable to investigate combined wing-empennage modes for high performance (V_D of 260 KEAS or above) airplanes, as well as for airplanes with unconventional configurations.

Flutter Analysis Evaluation: For a given set of input parameters, the resulting output generally consists of a number of theoretical damping values (g) with associated airspeeds and frequencies.

Various cross plots of these values among themselves and versus varied input parameters allow a study of trends. Common plots are: damping vs. equivalent airspeed (g-V plots), control surface balance vs. flutter speed, uncoupled frequency vs. flutter speed, altitude vs. flutter speed, etc. Normally only the critical items will be extensively compared.

Of particular importance is an evaluation in the neighborhood of the crossing of a damping velocity (g-V) curve toward the unstable damping region, through zero. The typical critical g-V curve will first become increasingly stable and with increasing speed will turn and rise toward or pass through g=0, then at some higher speed may again turn toward the stable region. Typical characteristics are discussed in the following examples:

<u>Examples</u>:



Curves 1 and 2 show slight trends toward instability, but do not approach actual instability.

Curve 3 crosses the stability axis but, depending on the inherent structural damping, may or may not actually become unstable. Curve 4 is obviously unstable and probably violent, since its slope is steep as it passes through zero. In actual flight it may only be a mile an hour or so between completely stable and extremely unstable explosive flutter. Flight tests are not advisable when this type plot is observed inside or near the flight envelope.

Much can be learned from g-V curves. (Absolute values should be viewed with some reserve as there is no perfect one-to-one correspondence of the analytical parameters and flight parameters.) Where the critical curve crosses the axis (with respect to V_D for the airplane) is important. Equally important is the rate of approach to instability (slope of curve).

The general practice is to use a damping value of g=0.03 at 1.2 V_D as the flutter limit of the g-V plots. However, this value should be used with caution if the slope of the curve is large (damping decreases very rapidly with an increase in airspeed) between g=0 and 0.03. In cases where the slope is steep, it is suggested that the g=0 airspeed be at least 1.2 V_D.

If flight flutter testing is conducted to verify damping under the above circumstances, extreme caution should be exercised.

For damping curves such as (3), which peak out below 1.2 V_D, the predicted damping should be no more unstable than g=0.02 unless justification is provided by other acceptable means.

5. ANALYSIS PLUS FLIGHT TEST. Although paragraph (c) of section 23.629 permits certification based upon flight test only, it is recommended that some analysis precede a flight flutter test. The results of any of the analysis procedures in paragraph 4 would be useful and could be used to provide guidance for formulating a flight flutter test plan. In all cases, as required by paragraph 23.629(a), the natural frequencies of main structural components should be determined by vibration tests or other approved methods prior to conducting any flight testing. A more thorough discussion of flight flutter testing is presented in appendix 3.

6. GROUND TESTING. Comparison of test data may be used in lieu of a totally new analysis in the case of dynamically similar aircraft. Comparison would usually be based upon geometry, mass and stiffness distributions, speed regime, and more importantly, upon a comparison of the measured coupled vibration modes.

a. Test data would normally include:

(1) Ground Vibration Testing
(2) Control Surfaces and Tab Mass Property Determination
(3) Stiffness Tests
(4) Free Play Measurement of All Tabs
(5) Rotational Frequency for All Tabs
(6) Tab System Rotational Stiffness

b. Appendix 1 presents some guidelines for recommended tests and procedures.

c. The degree of similarity between aircraft that is required for justification can vary greatly. Some of the factors which should be considered are the amount of safety margin available, flutter speed sensitivity to certain parameters, and the thoroughness of the original analysis.

7. WHIRL MODE. Beginning with Amendment 23-7, paragraph 23.629(e) required an investigation of the whirl mode phenomena for multiengine turbopropeller airplanes only. The basis being these airplanes characteristically have wing mounted engines wherein the stability of a flexibly mounted engine/propeller on an elastic wing is of major concern. Amendment 23-31 of paragraph 23.629(e) now requires an investigation of the whirl mode phenomena for both single and multiengine turbopropeller airplanes. Although airframe influence may be negligible for fuselage mounted single engine tractor configurations, the potential for propeller whirl flutter still exists. For pusher configurations, empennage motion may be significantly affected by engine/propeller forces. Stability of either installation is dictated, in part, by engine mount stiffness, damping, mass properties, motion axes, propeller geometry and propeller advance ratio. Therefore, to assure freedom from whirl mode flutter, all turbopropeller installation investigations should include, in addition to the appropriate airframe degrees of freedom:

a. Whirl mode degree of freedom which takes into account the stability of the plane of rotation of the propeller and significant elastic, inertial, and aerodynamic forces.

b. Propeller, engine, engine mount, and airplane structure stiffness and damping variations appropriate to the particular configuration; e.g., deterioration of engine isolators, large cantilevered engine installations, etc.

Generalized mathematics are presented in appendix 2. In addition, references 9, 10 and 11 of appendix 4 contain technical information for an acceptable means of demonstrating whirl mode stability.

CHAPTER 3. MODIFICATIONS TO AIRCRAFT ALREADY CERTIFICATED

8. REEVALUATION. Considerable judgment is often required to determine the degree of reevaluation necessary. If the mass, mass distribution, or the stiffness distribution are affected sufficiently to result in possible significant changes in resonant frequencies of major modes, mode shapes, or mass coupling terms in the flutter equations, then some reevaluation, such as pre-mod and post-mod GVT data comparison, or analysis may be required. Some examples of significant changes are:

a. Engine (Propeller). A change in mass or mass moment of inertia of the powerplant or in its mounting system (bushings, etc.) or a c.g. shift should be investigated. On single-engine airplanes, such changes will most likely affect fuselage and empennage frequencies and mode shapes. For engines mounted on the wings, the entire airplane may be affected.

For changes in existing designs which entail significant increases in engine power and/or airplane speed, special assessments of the effect on primary and secondary control systems should be made. If tabs are exposed to the propeller slip stream, particularly on airplanes with a dive speed greater than 260 KEAS, it may be necessary to impose the fail-safe criterion discussed in chapter 4.

b. Structural Cutouts. Severing or bridging across major structural members, such as fuselage bulkheads and ribs or stringers of aerodynamic surfaces, may produce discontinuities in stiffness parameters that significantly alter the vibratory response of the structure.

The significance of a change may be ascertained by its effect on the energy terms in the flutter modes being evaluated.

CHAPTER 4. CONTROL SURFACES AND TABS

9. RESPONSE. The aerodynamic force on an airfoil is very sensitive to control surface displacement, which in turn is responsive to both control motions and aerodynamic forces from tab displacement. Control surface displacement may result from deflection of the control system, deflection of the control surface attachment, or structural deflection of the control surface itself under forces from control application, aerodynamic force due to position or velocity of position change, and inertia force.

10. BALANCE. Control surfaces and tabs are balanced to prevent rotation about their hinges resulting from inertial response to motion in any flutter mode. When the flutter mode consists of motion about some axis perpendicular to the control surface hinge axis, a concentrated ballast is most efficiently used. Caution should be used to assure that its location is in a high response area of the vibratory mode, which is difficult when the mode is complex. Caution should also be used to assure that its attachment is secure. Because the attachment is subjected to oscillatory loads which cause fatigue failures and because a distributed ballast achieves balance against all flutter modes, it is conservative to distribute the ballast in accordance with the spanwise weight distribution of the surfaces. If less than static balance is provided, the effect of variations in the amount of balance should be evaluated. To guard against unintended balance changes in service, sealing and proper drain holes should be provided to minimize the risk of water, ice, or dirt accumulation in a control surface or tab. Excessive accumulation of these substances could alter the static and/or dynamic balance of the control sufficiently to adversely affect flutter characteristics.

11. VIBRATORY MODES. Control surface rotation about its hinge line is affected by various constraints. Control system stiffness and the rigidity of interconnection between control surfaces determine the primary rotational modes. Both symmetric and antisymmetric modes should be considered. Vibration mode changes resulting from the modifications to the control system such as the addition of a bob weight must be assessed for their effect on flutter. Secondary rotations may result from flexure of the attaching structure or bending of the control surface. This is a major consideration for long short-chord tabs and may affect their effective irreversible characteristics. When it is necessary to raise a tab frequency by redesign, consideration should be given to the contributions of: hinge bending perpendicular to the surface (especially near the horn-actuator station), horn length, axial stiffness of the push-pull rod or link, mounting flexibility and lateral stability at push-rod attachment of the tab actuating mechanism.

12. ANALYSES. In most cases involving control surfaces, the flutter speeds are largely governed by the mass balance values and distributions. It is wise for the flutter analyst to cover a range of balance values and distributions to determine the most satisfactory ones. It is common to find that a change which improves one mode degrades another. When conducting a

multi-degree-of-freedom analysis, it is advisable to investigate the effect of control system frequency from zero to about 1-1/2 times the system frequency measured in test. Due to friction, etc., it may be difficult to excite and measure control system frequency accurately. The stiffness can be measured at the surface with the control locked in the cockpit and, using the inertia of the end items, the system frequency can be calculated.

Theoretical values of tab and primary control surface aerodynamic derivatives have, for some configurations, produced higher flutter speeds than flutter model testing. Analytically derived tab and primary control surface aerodynamic coefficients based on strip theory have for some configurations produced higher flutter speeds than wind tunnel tests. Therefore, flutter speed sensitivity to variations in the theoretical coefficients should be evaluated in all control surface/tab investigations.

13. FAIL SAFE REQUIREMENTS. Amendment 23-23 of paragraph 23.629(f) requires flutter free operation after failure, malfunction, or disconnect of any single tab element. This fail-safe requirement is extended to include a failure, malfunction, or disconnect of any element in the primary flight control system or flutter damper on airplanes with a dive speed in excess of 260 KEAS below 14,000 feet, or MACH 0.6 above 14,000 feet.

Potential failures that require investigation include, but are not limited to, tab or primary control trim actuating system, primary control actuating system (both of which includes bellcranks, pulleys, brackets, and their attachments), and control cables or push rods. Control surface hinges and tab hinges, their attachments, and local portions of structure need not be included as part of the control system in this investigation.

Possible means of compliance to actuating system failures (i.e., actuators, cables, rods) may be achieved by incorporating dual systems, mass balancing the controls to counter the rotation of a zero stiffness free surface, or by incorporating a combination of the two. Proper mass balancing, particularly for tabs, requires considerable care and knowledge of the flutter mechanism to assure adequacy of the design in suppressing flutter. Dual load path designs should include an assessment of residual strength with a single failure to assure that the remaining path will not fail before the single failure is detected during appropriate specified inspection intervals.

CHAPTER 5. DIVERGENCE AND CONTROL REVERSAL

14. GENERAL. Steady state aeroelastic instabilities in an airfoil are avoided by providing adequate torsional rigidity. Methods to determine the adequacy of torsional rigidity are outlined in references 2 and 3 of appendix 4.

15. AIRFOIL DIVERGENCE. Divergence occurs when the aerodynamic torque exceeds the torque resisting capability of the wing. Because the aerodynamic torque is a function of speed as well as deflection, whereas the resisting torque is a function of deflection only, there exists a limiting divergence speed. Divergence may occur with no warning.

16. CONTROL REVERSAL. Control reversal will often be preceded by pilot comments of "heavy" or "sluggish" ailerons. A limiting reversal speed is reached when the change in lift due to control surface rotation is nullified by the change in lift due to airfoil twist.

APPENDIX 1. GROUND TESTING

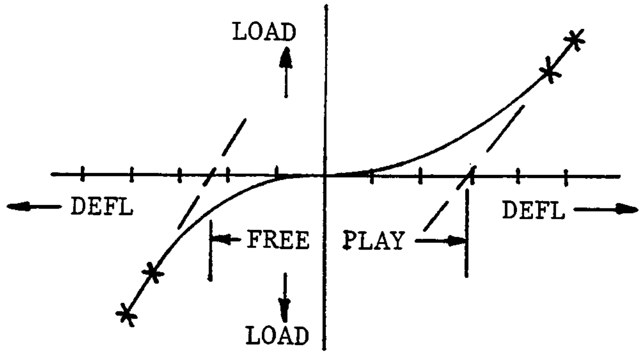
1. INTRODUCTION. The adequacy of the methods used to show compliance with section 23.629, as discussed in the main body of this document, is dependent upon the availability of reliable ground test data to verify the analytical data used and/or to serve as a basis for flutter substantiation per the simplified criteria of reference 1. This appendix, therefore, presents guidelines in conducting the more significant tests required to accomplish this objective. However, in keeping with the general purpose of this advisory circular, the information provided is not intended to be mandatory, nor is it to be considered an exhaustive treatment of the subject.

2. CONTROL SURFACE AND TAB MASS PROPERTIES. The experimental mass properties of control surfaces and tabs (weight, static moments, moments of inertia, and c.g.) are important ingredients in flutter substantiation. These properties form a basis for verification of the analytical data used in the rational analysis and provide the necessary parameters for use in the simplified criteria. Reference 1 presents a detailed procedure for the experimental determination of these properties.

3. TAB FREE PLAY. Free play tests provide the necessary data for determining the effectiveness of a tab in fulfilling the requirements for irreversibility as specified in the main body of this document. In addition to demonstrating the maximum free play available, these tests provide the stiffness of the actuating system for use in computing tab rotational frequency.

Free play and stiffness may best be measured by a simple static test wherein "upward" and "downward" (or "leftward" and "rightward") point forces are applied near the trailing edge of the tab at the spanwise attachment of the actuator (so as not to twist the tab). The control surface should be blocked to its main surface. Rotational deflection readings are then taken near the tab trailing edge using an appropriate measuring device, such as a dial gauge. Several stepwise load and deflection readings should be taken using loads first applied in one direction, then in the opposite.

A plot of these load deflections typically appears as follows:



Free play is then defined by extending the best straight lines through zero. System stiffness may then be obtained from the slopes of the curves away from the zero point.

4. INFLUENCE COEFFICIENT TESTS. Bending and/or torsion influence coefficient test results form the basis for the definition of component stiffness distributions. The extent of the tests depends on the intended use of the data. A full scale test program, wherein the coefficients of each spanwise mass strip are defined may be desired if experimental data is the primary source for defining component stiffness. In contrast, calculated influence coefficients, based on analytical bending (EI) and torsion (GJ) stiffness distributions, may be adjusted reliably with considerably less test data. A method is outlined below for determining influence coefficients for conventional structure, i.e., aspect ratio greater than four and unswept elastic axis.

The test article, wing, tailplane, or fin, is generally mounted at its root, without control surfaces, in a rigid test fixture for these tests. However, wing stiffness tests, particularly torsion as required for simplified criteria, may be successfully conducted with the wing mounted on the fuselage restrained in a cradle. This type of setup requires duplicate loading fixtures for right and left wing to balance the aircraft under load and thus minimize "jig rotation" effects.

The chordwise location of the elastic axis is determined by applying a torque load at selected stations and plotting the deflection vs. chord shear center or elastic axis at that station.

Torsional influence coefficients (radians twist about the elastic axis per unit torque load) are obtained by applying a pure torque load about the elastic axis at the tip and measuring the resulting spanwise twist. The twist per unit torque applied at intermediate inboard stations will be the same inboard of the load point. Thus, it is necessary to load only one additional inboard station, say 75% span, to check for data repeatability only. To insure that the load applied is a pure torque load, the deflections of the elastic axis should be monitored during the loading process. Zero deflections should result.

Bending influence coefficients (deflections per unit shear load) are obtained by applying shear load on the elastic axis at a selected station and measuring the resulting deflections at a sufficient number of spanwise locations to define the influence line for that load point. The procedure is repeated for each load station. To insure that the shear load is applied on the elastic axis, no appreciable chordwise variation in the measured deflections should be evident.

The experimental determination of fuselage stiffness properties can be accomplished essentially the same way as for the aerodynamic surfaces. In this test the fuselage is treated as two beams, forward and aft fuselage, each cantilevered from the wing-root attachment. It is extremely important that the fixture at this attachment be very rigid; and, any displacement of the test jig during loading must be monitored, regardless of how small, throughout the test for inclusion in the data analysis. Small displacements can be quite influential in a rather complex data reduction procedure, and if improperly done, can lead to erroneous and troublesome conclusions. On this basis it is often the practice to compute fuselage stiffness properties for the fuselage, then use ground vibration test results to tune calculated modes and, in turn, stiffness as required.

Thin-skinned structure may buckle at a very low load, reducing actual stiffness in flight considerably from that determined by the above procedure and the analyst is cautioned to investigate such conditions.

5. GROUND VIBRATION TESTS. Ground vibration testing has as its fundamental objective the definition of vibration mode frequencies, mode shapes, and damping characteristics of an aircraft. These data then become the basis for the analytical development of a mathematical vibration model of the airplane or serve as a check on such a model once it is developed. The results ultimately become the basis for rational flutter analyses. If the simplified flutter prevention criteria of reference 1, discussed in the main body of this advisory circular, is used, then the results from these tests are used directly to establish a predicted flutter speed of the airplane.

The degree of sophistication required to conduct a resonance test (techniques, recording equipment, suspension system, etc.,) depends upon the complexity of the structure being tested. Since it is impossible to cover all test situations that may arise, the discussions presented in this section are fundamental in nature, dealing specifically with sinusodial methods of excitations. They are intended as guidelines for those persons concerned with general type aircraft, who have only the basic test facilities. Other procedures employing random or impulse excitations are being used more frequently. However, these methods are considered beyond the scope of this AC.

a. Test Article and Suspension System. The airplane should be supported in a level attitude such that the rigid body frequencies of the airplane on its support are less than one-half the frequencies of the lowest elastic wing or fuselage mode to be excited.

One of the following methods of support can generally be used:

(1) Support the airplane on its landing gear with the tires deflated sufficiently to achieve the above result. Fifty percent normal tire pressure usually achieves good results. It may be necessary to block the landing gear struts to eliminate damping in the oleos.

(2) Suspend the airplane on springs.

(3) Support the airplane on its landing gear resting on spring platforms.

(4) Support the airplane fuselage and wings on large air-filled flotation bags.

The airplane should be equipped with all items having appreciable mass such as engines and tip tanks. The weight and c.g. of the test article should be determined to enable proper correlation with the math model. Where fuel is located in the outboard 50% of the wing semispan, it may be desirable to test a full fuel condition in addition to the empty condition in order to provide additional data for math model correlation.

It is generally advantageous to block the control surfaces in their neutral position when obtaining airframe modes.

b. Equipment. Various types of shakers are available, i.e., inertia, elastic, airjet, electromagnetic, etc. Electromagnetic exciters are generally preferred and most commonly used. This type consists of a coil that is attached to the structure with a fixed drive rod, as opposed to a flexible shaft or spring for inertia or elastic type shakers. The coil is surrounded by a magnetic field and is set in motion by an alternating current. Electronic oscillators and amplifiers are used to control this type of system.

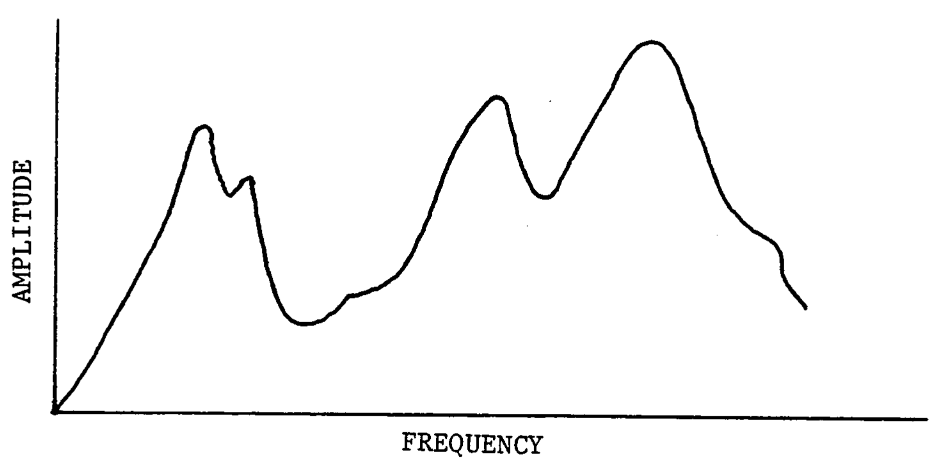
Vibration amplitude may be obtained by using either velocity pickups or accelerometers so long as transducer mass is insignificant. The output can be observed using a cathode ray oscilloscope and digital voltmeter. Phase relationship between two transducers can be noted with sufficient accuracy, and by exercising extra care, using an oscilloscope equipped with a grid screen.

Data systems are available that provide the coincident, in-phase or real term, and the quadrature, the imaginary term, responses of the total response frequency (the product of the force and reference signal). Graphical representation of these terms is presented, providing a very accurate identification technique for resonant frequencies and phase relationships. Structural damping is also readily available from these data.

Whatever data system is used, uniformity is recommended. Piecemeal systems, using velocity pickups and accelerometers, or filters with different characteristics, etc., can give erroneous data and should not be used without careful regard to their calibrations and performance characteristics and limitations.

c. General Procedures for Airframe Modes. It is usually sufficient to apply a harmonic excitation force to the structure provided the force is not applied in the proximity of a node line. For this reason vibrators are usually attached at an extremity such as the nose and/or rear of the fuselage or near the tips of the wing or empennage surfaces where nodes are not likely to occur.

With the shaker(s) and a reference pickup mounted at a selected location, frequency is varied upward through the range usually encountered in aircraft structures (2 to 100 H_z), With small increments of frequency, the response of the structure is recorded and the resulting plot of amplitude of response vs. forcing frequency is used to determine the resonant frequencies of the system. A typical sweep is shown below:



Although duplication of peak responses will result, it is advantageous to obtain frequency response records with a reference pickup positioned on each of the main surfaces and fuselage at a specific shaker location. This will reduce the chances of overlooking modes.

There are several criteria for establishing that the excited response approximates a normal mode of vibration. The most commonly accepted approach requires that all of the criteria below be met:

(1) A relative maximum response per unit input exists.

(2) Accelerations at all points in the structure are either exactly in phase or 180° out of phase with each other. The accelerations measured at all points on the structure during resonance will be either in phase or out of phase with a reference location but will be at a ± 90° phase angle with the force, for small values of damping.

(3) A decay record exhibits a single-frequency, non-beating, low-damped characteristic.

Having established the resonant frequencies, a survey of the aircraft is conducted with the shakers tuned to each frequency in-turn. A roving transducer is used to sense amplitude and phase angle relative to the reference pickup at each airplane location. An adequate number of points should be surveyed along the span and chord (typically on the spars) of each surface and along the fuselage to define the airplane modal displacements, and the associated node lines. To obtain proper phase relationship additional excitation may be necessary.

It may not be necessary to survey identical peak frequency responses although they occur at different locations. In all probability, the mode will be the same. This can be determined by checking only a few stations or simply by visually observing the motion of the aircraft.

Care should be exercised in defining component node lines for each mode. This is particularly important in evaluating the effectivity of balance weight locations.

d. Aircraft Structural Modes Usually Encountered. The modes excited during ground vibration depend on the type of configuration being tested. The vibration modes of an airplane that carries heavy mass on the wing, such as engines, tip tanks, etc., or has the stabilizer located high on the fin will be highly coupled and generally cannot be described except by diagrams that show the relative shape and phase of each part of the airplane. Airplanes that do not have these design characteristics usually have relatively uncoupled modes which can be described by naming the type of motion that is predominant. In general, the following predominant modes should be obtained insofar as is practicable.

(1) Wing Group Modes.

(i) For wings without engines, tip tanks, or heavy external or internal stores:

Wing vertical bending and wing torsion, fundamental and higher modes, symmetric and antisymmetric.

(ii) For wings carrying heavy masses outboard of the fuselage:

Wing bending coupled with wing torsion and flexible store (engines) modes, fundamental and higher modes, symmetric and antisymmetric.

(2) Fuselage - Empennage Group Modes.

(i) Fuselage Torsion (coupled with stabilizer antisymmetric bending).

(ii) Fuselage lateral bending and fin bending, fundamental and higher order consisting of two fundamental modes in which the fin tip and aft fuselage are in phase in one mode, and out of phase in the other.

(iii) Fin bending - symmetric and antisymmetric for multi-tail airplanes.

(iv) Fin torsion (generally highly coupled with stabilizer yawing if stabilizer is located at the outer span stations of the fin).

(v) Rudder bending and torsion.

(vi) Fuselage vertical bending and stabilizer bending, fundamental and higher order consisting of two fundamental modes in which the aft fuselage and stabilizer tips are in phase in one mode, and out of phase in the other.

(vii) Stabilizer torsion - symmetric and antisymmetric.

(viii) Stabilizer yawing for surface located at the outer span stations of the fin.

(ix) All movable horizontal tail - rotation coupled with bending, torsion.

(3) Engine or External Store Modes. For multiengine aircraft or aircraft carrying large pylon-mounted stores, the pitch, roll, yaw, and lateral and vertical translation modes should be defined. These modes should also be determined for all turbo propeller engine installations. It may be necessary to excite the engine fore and aft on the propeller blade to obtain the most critical pitch and yaw modes. If this method is used, consideration should be given to possible modal distortion due to propeller blade flexibility. Also, caution should be exercised and the engine manufacturer's instructions followed concerning possible damage to bearings when exciting the engine.

e. General Procedures for Control System Modes. The experimental determination of control surface and tab rotation modes about their hinge lines may be difficult due to inherent friction within the system or the masking of these modes by structural interaction. On this basis, extra care is required for proper identification of the system's characteristics.

For conventional aileron or elevator systems, the rotation modes may be successfully measured by applying a single excitation force to either the righthand or lefthand surface. However, multiple shakers are preferred, particularly if the right and left surfaces are operated from separate control systems. Likely shaker positions are on the trailing edge at midspan or on the horn leading edge. Tab rotation may be determined from the control surface excitation but usually a direct excitation on the tab surface is required with the control surface (aileron, elevator, or rudder) blocked to its main surface.

A transducer placed on the control being excited is used to monitor the response and determine peak frequencies by the same technique described for airframe modes. To define the modes excited, it is generally necessary to follow any or all of the following procedures:

(1) Monitor the phase between the right and left surface, the control column, or the attaching structure.

(2) Conduct a detailed survey of the surface, spanwise and chordwise, to define any structural modes. If the surface has a very long span or wide chord, these modes, bending and torsion, are likely to be dominant.

(3) Visually monitor the surface under excitation.

(4) Simple rationalization to distinguish the excited modes from previously defined airframe modes.

In the performance of these tests, the shakers and/or transducers may contribute sufficient weight to the surface being tested to significantly affect the frequency of the surface. This is particularly true for tabs with very small mass and rotational inertias. Dunkerly's equation, presented in reference 8, provides an acceptable method for correcting the measured frequency to the true surface frequency.

A check on experimentally determined modes may be facilitated by calculating rotational frequencies from measured inertias and system stiffness properties obtained from static tests.

For extremely light weight structures, another method that may be used to eliminate the shaker influence is to use an air shaker or other device which does not directly attach to the control surface or tab.

f. Control Surface Rotation. Symmetric aileron rotation, the normal opposed operational mode, with control stick fixed or free, is defined as the peak frequency at which both ailerons are rotating in phase. Antisymmetric rotation, the normal operation mode, generally has zero or very low stiffness.

Rudder rotation in the normal operation mode with pedals free occurs when the rudder and pedals are out of phase.

Elevator and all movable tailplane rotation modes should be determined with the pilot's controls fixed and free. Elevator rotation with the stick fixed is defined as the peak frequency at which both elevators are in phase for symmetric rotation, and out of phase for antisymmetric rotation. For all moving tailplanes or elevators with stability augmentation systems (control column bob weights and down springs), normal opposed operation with stick free will occur when the control stick and elevator are responding out of phase.

The effect of variations in control cable tension should be investigated.

g. Tab Rotation. Rotational modes for irreversible trim and servo tabs are determined experimentally to supplement the calculated frequency obtained from measured stiffness in the free play tests. Tab rotation frequency will usually vary with angular deflection and is determined at maximum trailing edge up, neutral, and maximum trailing edge down positions to determine the range of tab frequencies. For geared tabs, the rotation frequency is usually determined with the control surface at maximum deflections and at neutral.

Large tabs, either wide chord or very long with a single actuator, often tend to be difficult to measure in a resonance test. Wide chord tabs often become significantly involved with "plate modes" of their carrying surfaces, while long narrow tabs may have their lowest frequency in a torsional mode rather than rotation. On this basis, it may be necessary to survey each response frequency rather extensively to properly define each mode.

Test requirements for spring tabs are dependent upon the tab control system design. In general, the following tests should be conducted to provide the required data for a mathematical representation of a spring tab system. (These tests are similar to those discussed in the previous paragraph for all moving tailplane systems.)

(1) For a preloading spring, tests should be performed for several amplitudes including complete removal of the preload, if practical.

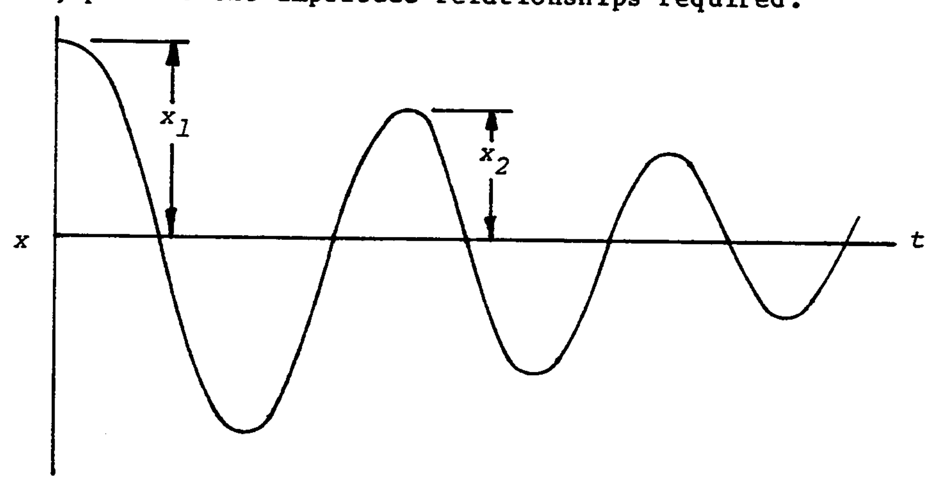
(2) Frequency of the control surface, with tab locked to surface and pilot's control column blocked, against the elastic restraint of the control system. A stick fixed mode.

(3) Frequency of the control column, with the control surface locked to its main surface, against the elastic restraint of the control system. A stick free mode.

(4) Frequency of the tab, with the control system cables disconnected and the control surface blocked to its supporting structure, against the elastic restraint of the springs in the tab system.

Spring loaded tabs are non-linear systems which are usually quite sensitive to small parameter changes making the design of these systems to preclude flutter most difficult. It is advisable to avoid their use unless extensive flutter analyses, including detail parameter evaluations, are conducted.

h. Structural Damping Measurements. Structural damping of each significant mode surveyed should be measured. The most commonly used procedure is based on the measurement of the rate of decay of oscillation. This is best expressed in terms of logarithmic decrement, the natural logarithm of the ratio of two successive amplitudes. Records of the response of a reference transducer, while driving the structure at a specific frequency and obtained immediately before and after power to the shaker is cut off, provide the amplitude relationships required.



The log decrement, δ, is then equal to ln (X_1/X_2); or as $\frac{0.693}{n}$ where n=no. of cycles to 1/2 amplitude; i.e.: $X_1/X_2 = 2$.

For small values of damping, the damping factor, γ or C/C_o, can be estimated as $\delta/2\pi$ and the structural damping $g = \delta/\pi$ (References 2 and 8).

i. Balance Weight Attachment. For control surfaces with balance weights mounted at one end of a cantilevered moment arm, the resonant frequency of the balance weight attachment arm should be at least 50% greater than the highest frequency of the fixed surface with which the control surface may couple. The control surface should be mounted in a jig and the vibrator attached to the balance weight. The input frequency is varied upward and the response of a reference transducer mounted on the balance weight is monitored to define the peak response.

All balance weight supporting structure should be designed for a limit static load of 24g normal to a plane containing the hinge and the weight, and 12g within that plane parallel with the hinge. The balance weight loads should be able to be carried by the control surface and by the fittings and their attachments on both sides of the hinge. Proof of these criteria can be accomplished by relatively simple static tests of the control surface mounted in a jig.

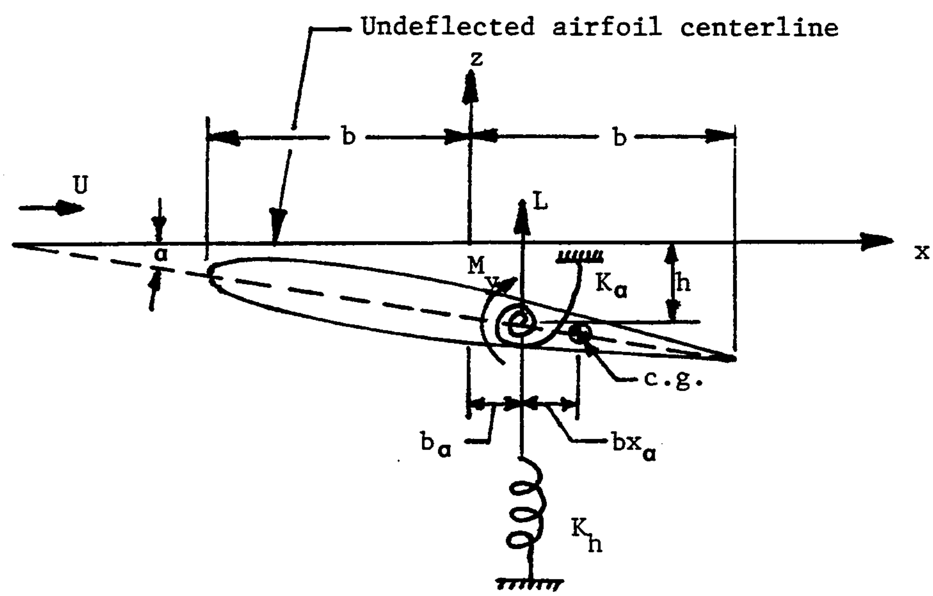
APPENDIX 2. FLUTTER ANALYSIS

1. INTRODUCTION. The objective of this appendix is to provide those persons responsible for predicting the flutter characteristics of Part 23 airplanes with some general guidelines for conducting a rational analysis. Two-Degree-of-Freedom, Three-Degree-of-Freedom, and Multi-Degree-of-Freedom Systems and Whirl Mode Analysis are considered briefly. The scheme of analyses outlined here makes use of uncoupled bending and torsion modes. This information herein should assist the analyst in determining the type of analysis suited to a given situation but is not sufficient to permit an analysis without a thorough study of the references.

Compressibility effects on the flutter speed should be considered at and above Mach 0.6.

2. TWO-DEGREES-OF-FREEDOM. The flutter characteristics of straight wings (or tails) of large aspect ratio can be predicted fairly well by considering a "Representative Airfoil" that has two-degrees-of-freedom, translation and pitch. This representative airfoil is usually given the geometric and inertial properties of the station three-quarters of the way from the centerline to the tip.

Information regarding this approach is contained in reference 3, and is outlined below:



Where: m = Mass of section

$S_\alpha = mbx_\alpha$ = Static Unbalance about Elastic Axis

I_α = Inertia about Elastic Axis

The equations of motion are:

$$\begin{bmatrix} m & S_\alpha \\ S_\alpha & I_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} m\omega_h^2 & 0 \\ 0 & I_\alpha\omega_\alpha^2 \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} Q_h \\ Q_\alpha \end{Bmatrix}$$

Assuming harmonic motion, one obtains:

$$-\omega^2 \begin{bmatrix} m & S_\alpha \\ S_\alpha & I_\alpha \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} + \begin{bmatrix} m\omega_h^2 & 0 \\ 0 & I_\alpha\omega_\alpha^2 \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -L \\ M_y \end{Bmatrix}$$

Using the aerodynamic expressions of Air Force Technical Report 4798, the expressions for L and M_y become :

$$L = -\pi\rho\, b^3\omega^2 \left\{ L_h \frac{h}{b} + [L_\alpha - L_h\ (1/2 + a)]\,\alpha \right\}$$

$$M_y = \pi\rho\, b^4\omega^2 \left\{ [M_h - L_h\ (1/2 + a)]\ \frac{h}{b} \right.$$
$$\left. + [M_\alpha - (L_\alpha + M_h)\ (1/2 + a) + L_h\ (1/2 + a)^2]\,\alpha \right\}$$

Where L_h, L_α, M_h, and M_α are functions of $V/b\omega$ and obtainable from reference 4.

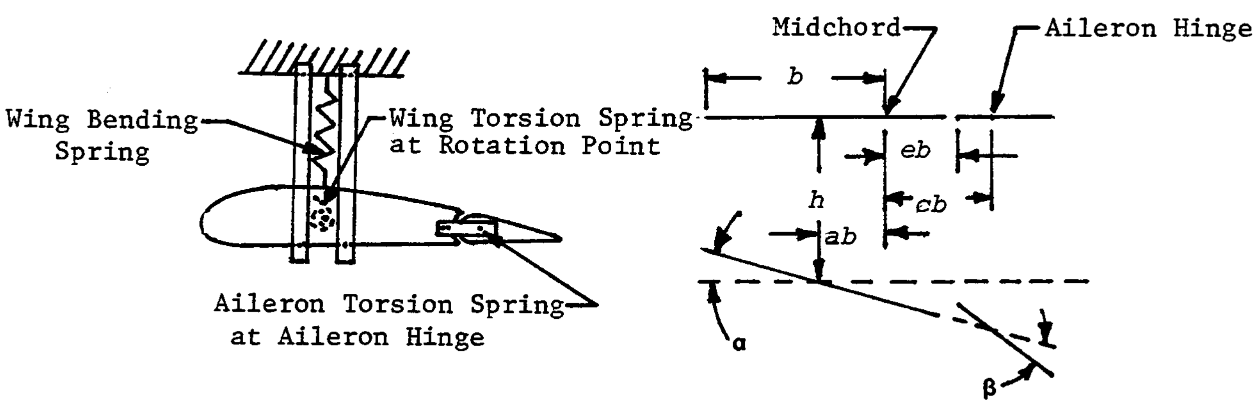
In this approach, the translation motions, h, are usually assumed to originate from the fundamental bending mode and the pitch motions, α, from the fundamental torsion mode. Higher modes or control surface rotations are not considered.

The value of velocity and damping obtained by solving the equations at each value of $V/b\omega$ are usually plotted to obtain the velocity at which the damping goes to zero.

3. <u>THREE-DEGREE-OF-FREEDOM</u>. Flutter mechanisms involving control surface rotation are many times more critical than those involving just bending-torsion. At least three degrees of freedom are required to analyze this phenomenon.

The procedure most commonly used is presented in references 2 and 4. The problem is first considered two-dimensional and then extended to the three-dimensional case.

a. <u>Two-Dimensional Flutter Theory</u>.



The motion of the system can be represented as above, where:

- b = semichord
- cb = distance between midchord and aileron hinge, positive if aft of midchord
- eb = distance between midchord and aileron leading edge, positive aft of midchord
- ab = distance between rotation point (elastic axis) and midchord
- h = bending deflection of rotation point (elastic axis), positive downward
- α = angular deflection about rotation point (elastic axis), positive for leading edge up
- β = angular deflection of aileron about aileron hinge relative to wing chord, positive for aileron leading edge up

Assuming linearized harmonic motion, the equations of motion become:

$$\overline{A}\,\frac{h}{b} + \overline{B}\alpha + \overline{C}\beta = 0$$

$$\overline{D}\,\frac{h}{b} + \overline{E}\alpha + \overline{F}\beta = 0$$

$$\overline{G}\,\frac{h}{b} + \overline{H}\alpha + \overline{I}\beta = 0$$

Where the coefficients are given in references 2 and 4. For each value of $V/b\omega$, the determinant of the coefficients matrix is set equal to zero and the resulting value of artificial damping plotted versus airspeed.

The limitations of a two-dimensional flutter theory are delineated in reference 4.

(1) All spanwise elements are considered identical with respect to all their flutter parameters.

(2) The vibration amplitudes in each mode do not vary with the spanwise location of the element under consideration.

(3) The effective aspect ratio approaches infinity.

(4) Aerodynamic flow over the oscillating airfoil is not disrupted by interferences.

In general, these limitations are prohibitive and some form of three-dimensional analysis is required.

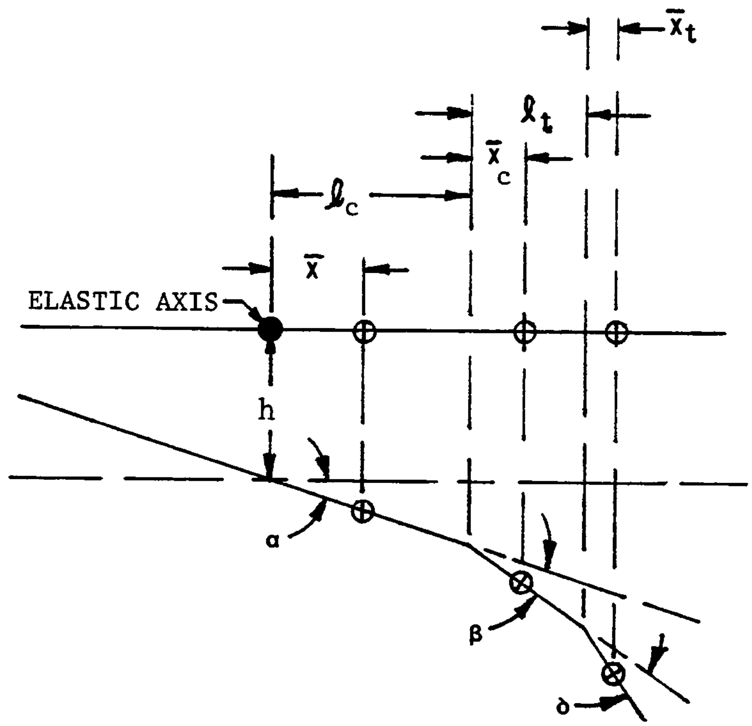
b. Three-Dimensional Flutter Theory. The typical three-dimensional analysis accounts for spanwise variations in mass, geometry and mode shape, but does not account for aspect ratio and aerodynamic interference effects. The mathematics is presented in Scanlan and Rosenbaum (ref.2). To use this approach one needs the spanwise distribution of the following parameters:

$m(x)$ = Mass per unit span of wing and aileron
$I_\alpha(x)$ = Mass moment of inertia per unit span of wing and aileron, referred to elastic axis
$S_\alpha(x)$ = Static unbalance per unit span of wing and aileron, referred to elastic axis
$I_\beta(x)$ = Aileron mass moment of inertia per unit span, referred to aileron hinge line
$S_\beta(x)$ = Aileron static unbalance per unit span, referred to hinge line
b = Semichord length
$(c-a)b$ = Distance between the elastic axis and aileron hinge line

The motion at flutter is assumed to be a superposition of the wing first uncoupled bending mode, the wing first uncoupled torsion mode, and the aileron rotation mode. In Scanlan and Rosenbaum the mode shapes are treated as continuous functions of span and integration performed over the span of the wing. The vibratory motion at flutter is considered unchanged by the aerodynamic forces.

In practice, the integration is performed numerically, rather than continuously. The wing is divided into a number of spanwise panels and approached from a lumped parameter concept.

4. <u>MULTI-DEGREE-OF-FREEDOM, THREE DIMENSIONAL FLUTTER THEORY</u>. It is not the intent of this section to present a detailed explanation, but rather to outline the general procedure for setting up the mathematics involved in a multi-degree-of-freedom flutter analysis. Familiarity with three-degree-of-freedom flutter analysis and matrix algebra is assumed. The system shown in the sketch is for each mass panel:



First, the general flutter equation with uncoupled modes is solved with the aerodynamic forces set to zero.

$$[\bar{M}]\{\ddot{q}\} + (1+ig)\lceil \bar{K} \rfloor \{q\} = [\bar{F}_A] = \text{zero}$$

$[\bar{M}] = [\Phi_u]^T[M] \lceil \Phi_u \rfloor$ Generalized mass (non diagonal)

$\lceil \bar{K} \rfloor = [\omega^2{}_u] \lceil \bar{M}_d \rfloor$ Generalized stiffness (diagonal)

Where: $\lceil \Phi_u \rfloor$ = Uncoupled assumed modal matrix

$\lceil \bar{M}_d \rfloor$ = diagonal elements of $[\bar{M}]$

$\lceil \omega^2{}_u \rfloor$ = uncoupled frequency matrix

$[M]$ = Mass Matrix

$\{q\}$ = Vector of generalized coordinates

The modal solution to this equation results in the coupled modes and the coupled frequencies which can now be used to solve the equation with airforces.

$$\lceil \bar{M} \rfloor \{\ddot{q}\} + (1+ig)\lceil \omega^2 \rfloor \lceil \bar{M} \rfloor \{q\} = [\bar{F}_A]$$

$\lceil \bar{M} \rfloor = [\Phi]^T[M][\Phi]$ = Generalized mass matrix (diagonal)

Where: $[\bar{F}_A]$ = Generalized aerodynamic forces

$[\Phi]$ = Coupled modal matrix

$\lceil \omega^2 \rfloor$ = Coupled frequency matrix (diagonal)

The equation of motion including the aerodynamic forces $\left\{ \bar{F}_A \right\}$ is solved for selected values of $\frac{V}{b\omega}$ to predict the damping in the same manner as previously discussed.

The $\left[M \right]$ matrix is a mass matrix, usually referred to the local elastic axis, and includes the necessary transformation terms for a control surface or tab. For each mass panel, i, the $\left[M \right]$ matrix is:

$$\begin{bmatrix} W & S\alpha & S\beta & S\delta \\ S\alpha & I\alpha & P\alpha\beta & P\alpha\delta \\ S\beta & P\alpha\beta & I\beta & P\beta\delta \\ S\delta & P\alpha\delta & P\beta\delta & I\delta \end{bmatrix}$$

Where:

$$W = W_W + W_c + W_t$$

$$S_\alpha = W_W \bar{x} + W_c (\ell_c + \bar{x}_c) + W_t (\ell_c + \ell_t + \bar{x}_t)$$

$$S_\beta = W_c \bar{x}_c + W_t (\ell_t + \bar{x}_t)$$

$$S_\delta = W_t \bar{x}_t$$

$$I_\alpha = W_W (\bar{x}^2 + k_w^2) + W_c \left[(\ell_c + \bar{x}_c)^2 + k_c^2 \right] + W_t \left[(\ell_c + \ell_t + \bar{x}_t)^2 + k_t^2 \right]$$

$$P_{\alpha\beta} = W_c \ell_c \bar{x}_c + W_t (\ell_t + \bar{x}_t) \ell_c + W_c (\bar{x}_c^2 + k_c^2) + W_t \left[(\ell_t + \bar{x}_t)^2 + k_t^2 \right]$$

$$P_{\alpha\delta} = W_t (\ell_c + \ell_t) \bar{x}_t + W_t (\bar{x}_t^2 + k_t^2)$$

$$I_\beta = W_c (\bar{x}_c^2 + k_c^2) + W_t \left[(\ell_t + \bar{x}_t)^2 + k_t^2 \right]$$

$$P_{\beta\delta} = W_t \ell_t \bar{x}_t + W_t (\bar{x}_t^2 + k_t^2)$$

$$I_\delta = W_t (\bar{x}_t^2 + k_t^2)$$

And:

W_W, W_C, W_t = Wts of primary surface, control surface, and tab, respectively.

$\overline{X}$ = Distance from surface CG to EA – Positive aft

$\overline{X}_c$ = Distance from control hinge to control CG

$\overline{X}_t$ = Distance from tab hinge to tab CG

ℓ_c = Distance from EA to control hinge

ℓ_t = Distance from control hinge to tab hinge

k_w, k_c, k_t = Radii of gyration of primary surface about its CG, control surface about its CG, and tab about its CG respectively.

The $\left[K\right]$ matrix is a matrix of stiffness influence coefficients. When referred to the elastic axis, the $\left[K\right]$ matrix for one mass panel becomes:

$$\left[K\right] = \begin{bmatrix} K^{hh} & K^{h\alpha} & K^{h\beta} & K^{h\delta} \\ K^{\alpha h} & K^{\alpha\alpha} & K^{\alpha\beta} & K^{\alpha\delta} \\ K^{\beta h} & K^{\beta\alpha} & K^{\beta\beta} & K^{\beta\delta} \\ K^{\delta h} & K^{\delta\alpha} & K^{\delta\beta} & K^{\delta\delta} \end{bmatrix}$$

Many times it is easier to actually measure flexibility influence coefficients, $\left[C\right]$, and then obtain $\left[K\right]$ by:

$$\left[K\right] = \left[C\right]^{-1}$$

The $\left[\phi\right]$ matrix is a mode shape matrix, the eigenvectors from solving the equation of motion without aerodynamics. For each mass panel

$$\left[\phi\right] = \begin{bmatrix} h_1 & h_2 & h_3 \cdots\cdots\cdots h_n \\ \alpha_1 & \alpha_2 & \alpha_3 \cdots\cdots\cdots \alpha_n \\ \beta_1 & \beta_2 & \beta_3 \cdots\cdots\cdots \beta_n \\ \delta_1 & \delta_2 & \delta_3 \cdots\cdots\cdots \delta_n \end{bmatrix}$$

Where n is the number of modes.

The $\{\bar{F}_A\}$ matrix represents the generalized aerodynamic forces, obtained from:

$$\{\bar{F}_A\} = \omega^2 F(\rho) [\phi]^T [AIC] [\phi] \{\bar{q}\}$$

Where $[AIC]$ represents the matrix of aerodynamic influence coefficients. There are several in use, but in general they will contain the following (as presented in Reference 5):

$$[AIC] = \begin{bmatrix} L_{hh} & L_{h\alpha} & L_{h\beta} & L_{h\delta} \\ M_{\alpha h} & M_{\alpha\alpha} & M_{\alpha\beta} & M_{\alpha\delta} \\ T_{\beta h} & T_{\beta\alpha} & T_{\beta\beta} & T_{\beta\delta} \\ Q_{\delta h} & Q_{\delta\alpha} & Q_{\delta\beta} & Q_{\delta\delta} \end{bmatrix}$$

These are referenced to the wing quarter chord, the control surface hinge line, and the tab hinge line. A transformation is required to relate these to the elastic axis.

Using the above explanations and assuming harmonic motion, the generalized equation of motion may be written for each coupled frequency as:

$$\left\{-\omega^2 [\bar{M}] + (1 + i g) [\bar{K}]\right\}\{\bar{q}\} = \omega^2 F(\rho) [\bar{A}] \{\bar{q}\}$$

Where:

$q = \bar{q} e^{i \omega t}$

ω = Harmonic frequency

$F(\rho)$ = A scalar function of density

$$[\bar{A}] = [\phi]^T [AIC] [\phi]$$

Rearranging the flutter equation, we may write

$$\left\{[\bar{M}] + F(\rho)[\bar{A}]\right\}\{\bar{q}\} - \lambda [\bar{K}] \{\bar{q}\} = \{0\}$$

Where the eigenvalue λ is defined by

$$\lambda = \frac{1 + i g}{\omega^2} = \underset{\text{(Real Part)}}{\operatorname{Re} \lambda} + i \underset{\text{(Imaginary Part)}}{\operatorname{Im} \lambda}$$

Hence the flutter frequency becomes

$$\omega_f = \sqrt{\frac{1}{Re\ \lambda}}$$

And the damping g

$$g = \omega^2\ Im\lambda = \frac{Im\ \lambda}{Re\ \lambda}$$

And the flutter speed is defined by:

$$V_f = \frac{\omega_f\ b_r}{k_r}$$

Where the equations have been non-dimensionalized and b_r equals a reference length and k_r equals a reference value of reduced frequency, $\frac{\omega b}{V}$.

5. WHIRL MODE.

It is the intent of this section to outline the general procedure for setting up the mathematics involved in a whirl mode analysis. Familiarity with whirl mode analysis is assumed. This analysis includes only engine cantilevered modes. Coupling of these modes with a flexible airplane's modes may be accomplished by the superposition of modes using the method outlined in paragraph 4.

The whirl mode stability equation for cantilevered power package is:

$$p^2 \left[FM\right] + p\left[FD + FG + FQ2\right] + \left[FK + FQ1\right] = 0$$

where $FM = \left\lceil GM(M) \right\rfloor$, generalized masses (lb. sec.^{2}/in.)

$$FK = (2\pi)^2 \left\lceil (FREQ(M))^2 \right\rfloor \left\lceil GM(M) \right\rfloor,$$
generalized stiffnesses (lb./in.)

$$FD = (.02)\ (2\pi) \left\lceil FREQ(M) \right\rfloor \left\lceil GM(M) \right\rfloor,$$
generalized damping (lb. sec./in.)
(.02 is the assumed structural damping)

$$FG = \left[PHI(I,M)\right]^T \left[G\right] \left[PHI(I,M)\right]$$

$$FQ_2 = RAD*V*\left[PHI(I,M)\right]^T \left[Q2\right] \left[PHI(I,M)\right]$$

$$FQ_1 = RAD*V^2*\left[PHI(I,M)\right]^T \left[Q1\right] \left[PHI(I,M)\right]$$

M = Number of modes
FREQ(M) = Frequency (Hz)
PHI(I,M) = Prop displacement matrix
RAD = Relative Density Ratio, ρ/ρ_o

NOTE: A special purpose computer program may be required for each power package to determine its vibration mode shapes and frequencies and the generalized masses for each mode.

The equation for gyroscopic forces is shown below.

GYROSCOPIC MATRIX - G

$$\begin{Bmatrix} F_{Z_P} \\ F_{\theta_P} \\ F_{Y_P} \\ F_{\psi_P} \end{Bmatrix} = \begin{bmatrix} & & & \\ & & & I_p\Omega \\ & & & \\ & -I_p\Omega & & \end{bmatrix} \begin{Bmatrix} \dot{Z}_P \\ \dot{\theta}_P \\ \dot{Y}_P \\ \dot{\psi}_P \end{Bmatrix}$$

F_{Z_P} = Prop. vertical force

F_{θ_P} = Prop. pitching moment

F_{Y_P} = Prop. side force

F_{ψ_P} = Prop. yawing moment

Z_P = Prop. vertical displacement

θ_P = Prop. pitch angle

Y_P = Prop. lateral displacement

ψ_P = Prop. yaw angle

Ω = Rotational velocity (radians/sec.)

I_P = Prop. polar mass moment of inertia (lb. sec.^{2} in.)

The prop aerodynamic force equation for each set of prop coefficients are shown below.

PROP AERODYNAMIC MATRIX - Q2

$$\begin{Bmatrix} F_{Z_P} \\ F_{\theta_P} \\ F_{Y_P} \\ F_{\psi_P} \end{Bmatrix} = \frac{\rho V}{\rho_o} \begin{bmatrix} -\rho_o C_{Z\dot{\theta}} \frac{S}{2} & & -\rho_o C_{Z\psi} \frac{S}{2} & \\ -\rho_o C_{\theta\theta} \frac{Sd}{2} & -\rho_o C_{\theta\dot{\theta}} \frac{Sd^2}{2} & -\rho_o C_{\theta\psi} \frac{Sd}{2} & \\ -\rho_o C_{Y\theta} \frac{S}{2} & & -\rho_o C_{Y\psi} \frac{S}{2} & \\ -\rho_o C_{\psi\theta} \frac{Sd}{2} & & -\rho_o C_{\psi\psi} \frac{Sd}{2} & -\rho_o C_{\psi\dot{\psi}} \frac{Sd^2}{2} \end{bmatrix} \begin{Bmatrix} \dot{Z}_P \\ \dot{\theta}_P \\ \dot{Y}_P \\ \dot{\psi}_P \end{Bmatrix}$$

PROP AERODYNAMIC MATRIX - Q1

$$\begin{Bmatrix} F_{Z_P} \\ F_{\theta_P} \\ F_{Y_P} \\ F_{\psi_P} \end{Bmatrix} = \frac{\rho V^2}{\rho_o} \begin{bmatrix} & -\rho_o C_{Z\theta} \frac{S}{2} & & -\rho_o C_{Z\psi} \frac{S}{2} \\ & -\rho_o C_{\theta\theta} \frac{Sd}{2} & & -\rho_o C_{\theta\psi} \frac{Sd}{2} \\ & -\rho_o C_{Y\theta} \frac{S}{2} & & -\rho_o C_{Y\psi} \frac{S}{2} \\ & -\rho_o C_{\psi\theta} \frac{Sd}{2} & & -\rho_o C_{\psi\psi} \frac{Sd}{2} \end{bmatrix} \begin{Bmatrix} Z_P \\ \theta_P \\ Y_P \\ \psi_P \end{Bmatrix}$$

where S = propeller disk area (sq. in.)

d = propeller diameter (in.)

ρ_o = air density at sea level (lb. sec.^{2}/in.^{4})

ρ / ρ_o = relative air density

V = true air speed (in./sec.)

In terms of the basic propeller aerodynamic derivative defined in reference 13:

Vertical force derivative due to pitch angle

$$C_{Z\theta} = -C_{Y\psi} = \text{CZTH}$$

Pitching moment derivative due to pitch rate

$$C_{m_q} = C_{\theta\dot{\theta}} = C_{\psi\dot{\psi}} = \text{CMQ}$$

Pitching moment derivative due to yaw angle

$$C_{m\psi} = C_{\theta\psi} = -C_{\psi\theta} = \text{CMPSI}$$

Pitching moment derivative due to pitch angle

$$C_{m\theta} = C_{\theta\theta} = C_{\psi\psi} = \text{CMTH}$$

Vertical force derivative due to yaw angle

$$C_{Z\psi} = C_{Y\theta} = \text{CZPSI}$$

The propeller coefficients must be furnished in a comparable form to those found in reference 13. For each true air speed, the coefficients are found as a function of prop blade angle determined from the advance ratio (forward velocity divided by prop tip velocity).

The solution to the whirl mode stability equation is found as shown below.

(1) Summing the matrix coefficients lets the equation read:

$$p^2 A + pB + C = 0$$

(2) Inverting matrix A and premultiplying the equation by A^{-1} yields:

$$p^2 - p\,[H1] - [H2] = 0$$

where $[H1] = -A^{-1}B$

$[H2] = -A^{-1}C$

(3) If we let an arbitrary vector, x_1, postmultiply each term

$$p^2 x_1 - p \left[H1\right] x_1 - \left[H2\right] x_1 = 0.$$

(4) Now, let

$$\left[H2\right] x_1 = p\, x_2 .$$

(5) Then, by substitution,

$$p^2 x_1 - p \left[H1\right] x_1 - p\, x_2 = 0.$$

(6) Dividing by p, the above equation becomes

$$p\, x_1 - \left[H1\right] x_1 - x_2 = 0.$$

(7) Rearranging terms in (6) and combining with equation (4) in a single equation of twice the order, we obtain

$$\left[\begin{array}{c|c} H1 & I \\ \hline H2 & \end{array}\right] \left\{\begin{array}{c} x_1 \\ \hline x_2 \end{array}\right\} = p \left\{\begin{array}{c} x_1 \\ \hline x_2 \end{array}\right\}$$

(8) The above equation is solved, using the Laguerre method described in Reference 14, for its eigenvalues, p, which come in complex conjugate pairs. The frequency of the whirl mode is determined by

$$f = \frac{1}{2\pi} \quad p_{IMAG} \quad ;$$

hence, the roots with negative imaginary values are discarded. The damping required to produce instability is determined by

$$g = 2p_{REAL}/(p_{REAL}^2 + p_{IMAG}^2)^{.5} \quad ;$$

where positive values are unstable.

(9) The eigenvector is found by substituting the eigenvalue solution in (1) and combining terms and postmultiplying by y to obtain

$$Dy = 0$$

(10) Then, by rearranging the first row

$$y(1) = \left[D(1,1)\right]^{-1} \left[-D(1,J)\ y(J)\right]_{J = 2,N}$$

where N = the number of modes.

(11) And by rearranging the remaining rows

$$\left[D(I,J)\right] \left\{y(J)\right\} = \left\{-D(J,1)\right\} y(1)$$

where I and J go from 2 to N.

(12) If we let y(1) = (1.0,0), recalling that it is complex, then

$$\left\{y(I)\right\} = -\left[D(I,J)\right]^{-1} \left\{D(J,1)\right\}$$

where I and J go from 2 to N.

(13) The mode shape is then determined by

$$PHIWM(L) = \sum_{I=1,N} (PHI(L,I)) * y(I).$$

(14) This complex vector is normalized so that the largest translation (L=1 or 3) has an amplitude of unity with the phase relationship preserved.

APPENDIX 3. FLIGHT FLUTTER TESTING

1. INTRODUCTION. This appendix presents a general discussion of acceptable procedures for conducting flight flutter tests intended as final proof of flutter free operation for new or modified airplanes. The methods described herein do not represent a comprehensive survey of existing techniques, but rather represent methods which have been proven to be particularly adaptable to general aviation aircraft.

Paragraph 23.629(c) permits the use of flight tests as the only means of showing freedom from flutter. However, it is recommended that these tests be conducted only after appropriate analyses, defining the critical conditions and severity of flutter onset, have been performed. Both the risk and scope of testing required to substantiate the total airplane is significantly increased without the benefit of reference analyses. In-flight excitation of only the critical mode(s) is, generally, all that is necessary for final demonstration of flutter free operation if preceded by rational flutter analyses. However, without these studies, all modes of the airplane, or those modes affected by the modification of an altered airplane, must be excited in flight. All test airplanes, whether the objective is to support analyses or to serve as the singular means of flutter substantiation, should include proper instrumentation for recording airplane response.

2. DETERMINATION OF VIBRATION CHARACTERISTICS. Paragraph 23.629(a) requires a determination of the natural frequencies of main structural components by vibration tests or other approved methods. This must be done regardless of the flutter substantiation method selected; i.e., (a) rational analysis, (b) flight flutter tests, (c) simplified criteria, or (d) combinations of these. This determination must be made for all new airplanes and for existing airplanes, before and after any major modification to assess the effects of these structural changes. Engineering judgement should be exercised in determining whether the effects of the modification on aerodynamics, stiffness, or mass are sufficient to warrant flutter reinvestigation. The effects of variations in fuel loading, airplane weight, and center of gravity should also be assessed. It is recommended that mode shapes, as well as frequency, be determined by either ground vibration tests or analytical methods, if adequately supported by test.

For modified airplanes with no available analyses, the degree of frequency change requiring flight investigation is dependent upon, in part, the nature of the modification, the relative change of bending-torsion frequency ratios, and the relationship of the structural modes to control surface modes. Shifts in node lines may also dictate flight checks.

3. AIRCRAFT EXCITATION METHODS. The airframe modes and frequencies can be excited in flight by any number of techniques. The important criteria for technique selection is that the modes and frequencies of interest must be

adequately excited to allow for proper modal response. To illustrate, most general aviation aircraft use cable or push rod control systems which have high levels of coulomb damping. The coulomb damping will cause a non-linear control response. At low amplitudes the damping will be high and the system stable; whereas, at higher amplitudes, the coulomb damping will be reduced and the control system could be unstable. A proper level of modal excitation will, therefore, produce lower system damping and an earlier indication of a developing flutter mechanism. Consequently, without proper excitation, the test engineer may have very little warning of developing flutter.

Consideration should also be given to the possible influence of the excitation system on the flutter characteristics of the airplane.

Excitation methods presently being used for general aviation airplanes are discussed briefly in the following paragraphs.

a. PILOT INDUCED CONTROL SURFACE IMPULSES. Pulsing the controls may be used to excite modes, generally, below 10 Hz, but is not recommended above 10 Hz. The effectiveness of the method is usually limited by either the ability of the pilot to impart a pulse of proper duration or the ability of the control to transmit the pulse to the primary surface. Three degrees of control rotation are normally sufficient if the duration is short enough to encompass all harmonics of interest.

b. SINUSOIDAL EXCITATION USING ROTATION MASSES. This technique has been used successfully for exciting airplane modes between 10 and 50 Hz. Rotating eccentric mass shakers mounted in the wing tips and/or the tail section of the fuselage will usually produce wing and empennage modes of concern. Although shakers in each wing tip and both vertical and lateral mounted fuselage shakers may be required to excite both symmetric and anti-symmetric modes, single shakers have been used successfully for both symmetries where adequate frequency separation exists. The eccentric mass should be large enough to excite the 10 Hz modes and the shaker supporting structure strong enough to withstand the shaker force at 50 Hz. Shaker forces of up to 300 pounds at 50 Hz have been used and produced very good results. In general, the larger the exciting forces the safer the test since the non-linearities will be minimized. The shaker force should be adequate to assure that modal response is easily distinguishable from random or buffet excitation.

A simple inertial shaker system can be constructed using off-the-shelf shop components. The system consists of a container of compressed nitrogen, pressure regulator, line gate valve, air hose and shop drill motor. The RPM of the drill motor is controlled by varying the line pressure and can be monitored by a tach generator attached to the drill motor shaft. The only parts requiring design and fabrication are the eccentric mass and appropriate mounting brackets.

c. EXCITATION USING AUTOPILOT AND OTHER METHODS. Sinusoidal excitation using the autopilot will produce modal responses similar to the inertial system but has the advantage of supplying a stronger input to the fundamental modes. A restriction is its ability to transmit energy into the control surfaces at the higher frequencies due to control system flexibility.

Other methods such as flutter vanes and rocket impulse units may be used. Regardless of the method used, the same principles of frequency response and mode identification apply; i.e., adequate response of the modes and frequencies will produce the desired indication of any developing flutter mechanism.

4. AIRCRAFT INSTRUMENTATION. The aircraft instrumentation required to adequately monitor vibration characteristics will vary greatly depending on the extent of the test (number of modes being investigated) and the special design characteristics. As a minimum, transducers that measure acceleration or velocity should be installed on the tips of the aerodynamic surfaces of concern; i.e., for wing and horizontal stabilizers, front and rear spars on one side for bending and torsion response and on one spar of the opposite side as an indicator of symmetry. The frequency response characteristcs of the installed transducer should be checked to assure adequate sensitivity throughout the test frequency range. Strain gages or accelerometers should also be installed on each control surface. Care should be taken to assure control balance is not disturbed by the instrumentation.

Unless workload makes it prohibitive, it is preferred that the in-flight data recorder and exciters be operable by the pilot to eliminate any need for additional personnel on board the airplane during the test. This is particularly desirable if testing is performed without supporting analysis.

Although various telemetry systems exist for recording and transmitting data to the ground, an on-board oscillograph or magnetic tape recorder are more commonly used. A magnetic tape recorder is advantageous over an oscillograph if filtering, sensitivity change, and speed variations are available on playback to aid in determining frequency and damping. An oscilloscope, or other means to monitor input frequency in real time, should also be available to the crew.

The complete excitation and data recording system should be thoroughly checked and calibrated, on the ground and as mounted in the airplane, to assure excitation of the desired modes and in turn to establish baseline amplitude and damping data. The response to engine noise and aerodynamic buffeting can seriously distort the data and, as a general rule, the signal-to-noise ratio should be at least 4 to 1. The unwanted signals can be filtered or minimized by increasing the exciter force. However, the maximum level of excitation should be limited to prevent structural damage from dynamic overload.

5. TEST CONDITIONS. Flight flutter tests should be conducted with the airplane configured to provide maximum safety to the crew. In preparing the airplane, a checklist of configuration requirements should be closely adhered to. The list should include such items as:

a. The airspeed indicating system should be calibrated.

b. All equipment items in the cabin should be secured adequately to meet emergency landing load requirements of section 23.561.

c. When certification is by FAR 23.629(c) alone, control surfaces should be balanced to the most under balanced (tail heavy) condition and trim tab free play set at the maximum allowed. If certification is by analysis and testing, the analysis should dictate the appropriate settings.

d. Control system damping should be minimized to simulate wear.

e. The crew should be provided with parachutes.

f. Each of the crew members should have easy access to an escape exit.

g. Each emergency exit door should be equipped with a quick release mechanism allowing the door to separate from the airplane. The doors should be checked at various airplane yaw angles to make sure that the pressure distribution over the door will allow it to be drawn away from the airplane.

h. For airplanes with large cabins, a knotted rope should be installed the length of the cabin.

The airplane configuration(s) to be tested (i.e., fuel loading, c.g., weight) obviously depends upon the purpose of the test, upon whether the airplane is a new type design or a modified type design, upon the nature of the modification if the airplane is a modified type design, etc. Generally, a minimum of two wing fuel loading conditions, representing maximum and minimum, should be checked if this parameter effects a significant change in component modes. Acceptable tolerances from the selected condition, to account for fuel usage, must be established and maintained throughout the tests. The need to explore airplane weight and c.g. variations must be based on the effect of these parameters on the airplane component(s) of concern.

At least two altitudes should be checked, if appropriate to the design. One representative of high dynamic pressure (approximately 9,000 feet will provide emergency egress protection) and one at approximately 75% service ceiling or the altitude at the V_d/M_d knee, if a design limiting condition.

6. FLIGHT TEST PROCEDURES. Due to the potential dangers involved in conducting flight flutter tests, there is always the desire of the crew to get the tests over with and a general tendency to want to skip speed points and short cut the test. For this reason, a flight plan should be established prior to beginning the test and followed as closely as possible.

A chase plane should be used for all tests.

Testing should begin at a low airspeed point to establish a data base; i.e., frequency correlation with zero velocity modes. Follow-on increments should be based on the flight history of the airplane. If the airplane has previously flown to some limit speed, then enough test points should be checked to that pre-achieved speed to develop a data trend. For unexplored speeds, the increments should be smaller and no more than two speed data points checked at any one altitude during a flight unless data trends show continuously increasing, or very high, damping with no indications of ensuing shifts in this trend. More data points may be checked per flight if telemetry and real time data analysis systems are available. Between V_C and V_D, the number of airspeed increments should be increased.

Atmospheric turbulence should be avoided as much as possible during these tests to eliminate superposition of unwanted random signals on the data records and to preclude possible structural overload at speeds near V_D.

If inertia shakers, aero vanes, or other sinusoidal exciters are used, frequency sweeps should be conducted at each speed point. It is also recommended that shaker dwells be performed at selected airspeeds as a check on the damping characteristics established from sweeps. These are conducted by tuning the shaker(s) to each peak frequency, allowing time for the airplane to stabilize, then cutting the shaker power and recording the response of each transducer. If pilot induced control impulses are used, each axis should be pulsed at least twice in each direction wherein the airplane is allowed to stabilize, with hands off controls, prior to the next impulse or speed point.

7. DATA REDUCTION AND INTERPRETATION. The methods used for analyzing flight flutter data will depend on (1) the type of excitation used, (2) the availability of electronic analysis equipment, (3) the degree of accuracy required, and (4) the time allowed for data reduction. Generally, absolute damping values are not necessary to achieve the objective of the flight tests wherein monitoring of damping versus airspeed trends is the primary concern. The approximation methods addressed briefly in this section will allow development of reliable trends provided consistent procedures are followed in reducing the response traces throughout the test.

When impulse excitation is used, the damping can be obtained by measuring the decay rate directly from the response traces. For cases where several frequencies are being excited by the impulse, it may be necessary to reverse the transient and play it into a tuned filter. The frequency of the tuned filter can then be varied to yield discrete modal responses (reference 7).

When continuously forced oscillation techniques are used to excite the structure, either the amplitude response method or the vectorial analysis method as developed by Kennedy and Pancu (reference 6) can be used to reduce the data. The amplitude response method is advantageous for general aviation applications since it provides a good approximate damping level, requires a minimum of electronic equipment, and the data can be quickly reduced. For this method it is assumed that the relative damping ratio is approximately inversely proportional to the maximum resonant amplitude of the respective modes. Therefore, if the amplitude for a given mode is increasing as the airspeeds increase, a reduction in stability will be indicated. The approximation method is briefly outlined as follows:

a. The product of response amplitude times damping will be constant for a given exciter force; i.e., $(A_g)\ (g_g) = c$

b. If the exciter force relative to frequency remains consistent, then at a given airspeed, the net structural damping will be $(A_v)\ (g_v) = c = (A_g)\ (g_g)$
then:

$$g_v = (A_g)(g_g)/A_v$$

where:
A_g = peak amplitude on ground
A_v = peak amplitude in air at test velocity
g_g = damping measured on ground at zero velocity
g_v = net structural damping at test velocity

c. The net damping (g_v) can be defined as the actual structural damping measured on the ground (g_g) minus the analytical aerodynamic damping (g) determined from the flutter analysis with zero structural damping; i.e., $g_v = g_g - g$, where g is negative when stable.

d. To permit use of this method, the shaker force and sweep rate must remain consistent from run to run.

A gradual increase in peak response (A_v) by a factor of 3 or a rapid increase in A_v by a factor of 2 will normally require stopping the test to inspect the airplane, instrumentation, etc. It is also possible to experience a very sharp rise in damping followed by a sharp decrease leading to violent flutter, thus making it difficult to predict trends without the aid of reliable analyses.

It is advisable to check the effects of variation in sweep frequency on the peak response damping using techniques included in reference 2. It is also advantageous to supplement continuously forced damping measurements with damping obtained from decay records. Flutter margin predictions per the techniques of Zimmerman and Weissenburger (Journal of Aircraft, 1964, Volume 1, Number 4) may in turn be a beneficial data presentation approach to supplement the characteristic velocity versus damping plots.

APPENDIX 4. ACKNOWLEDGMENTS, REFERENCES AND BIBLIOGRAPHY

ACKNOWLEDGMENTS:

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